Online appendix to *International trade with an oligopolistic transport sector*[*]

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November 2018

Demand for transport

In contrast to the main analysis, this section describes demand for shipping as a function of freight rates, and then inverts this demand system for a well–defined Cournot game. While independent of the main analysis, I include this dual approach for the sake of completeness.

Let $\boldsymbol{t}_{\cdot d} \equiv (t_{1d}, \ldots, t_{Od})$ be the vector of freight rates along routes destined for d, and abbreviate demand for shipping along route (o, d) by a manufacturer with unit labour requirement a, given freight rates $\boldsymbol{t}_{\cdot d}$ to

$$s^{*}_{od}\left(a \middle| p^{\max}_{d}\left(\boldsymbol{t}_{\cdot d}\right), t_{od}\right) = \frac{L_d}{2\gamma}\left[p^{\max}_{d}\left(\boldsymbol{t}_{\cdot d}\right) - C_{od}\left(a, t_{od}\right)\right] \equiv s^{*}_{od}\left(a \middle| \boldsymbol{t}_{\cdot d}\right). \tag{0.1}$$

Similarly, let

$$\hat{a}^{*}_{od}\left(\boldsymbol{t}_{\cdot d}\right) \equiv \hat{a}_{od}\left(p^{\max}\left(\boldsymbol{t}_{\cdot d}\right), t_{od}\right), \qquad \overline{a}_{od}\left(\boldsymbol{t}_{\cdot d}\right) \equiv \mathbb{E}\left[a \middle| a \leq \hat{a}^{*}_{od}\left(\boldsymbol{t}_{\cdot d}\right)\right] \tag{0.2}$$

denote unit labour requirements of the marginal and average o–based manufacturers active in d, with corresponding demands $\hat{s}^{*}_{od}\left(\boldsymbol{t}_{\cdot d}\right) \equiv s^{*}_{od}\left(\hat{a}_{od}\left(\boldsymbol{t}_{\cdot d}\right) \middle| \boldsymbol{t}_{\cdot d}\right)$ and $\overline{s}^{*}_{od}\left(\boldsymbol{t}_{\cdot d}\right) \equiv s^{*}_{od}\left(\overline{a}_{od}\left(\boldsymbol{t}_{\cdot d}\right) \middle| \boldsymbol{t}_{\cdot d}\right)$.

Aggregate demand for shipping along lane (o, d) is

$$S_{od}\left(\boldsymbol{t}_{\cdot d}\right) = \int_{0}^{\hat{a}^{*}_{od}\left(\boldsymbol{t}_{\cdot d}\right)} s^{*}_{od}\left(a \middle| \boldsymbol{t}_{\cdot d}\right) \cdot L_o \, \mathrm{d}G_o\left(a\right). \tag{0.3}$$

The choke price, written here as a function of freight rates, is the solution to the fixed point problem

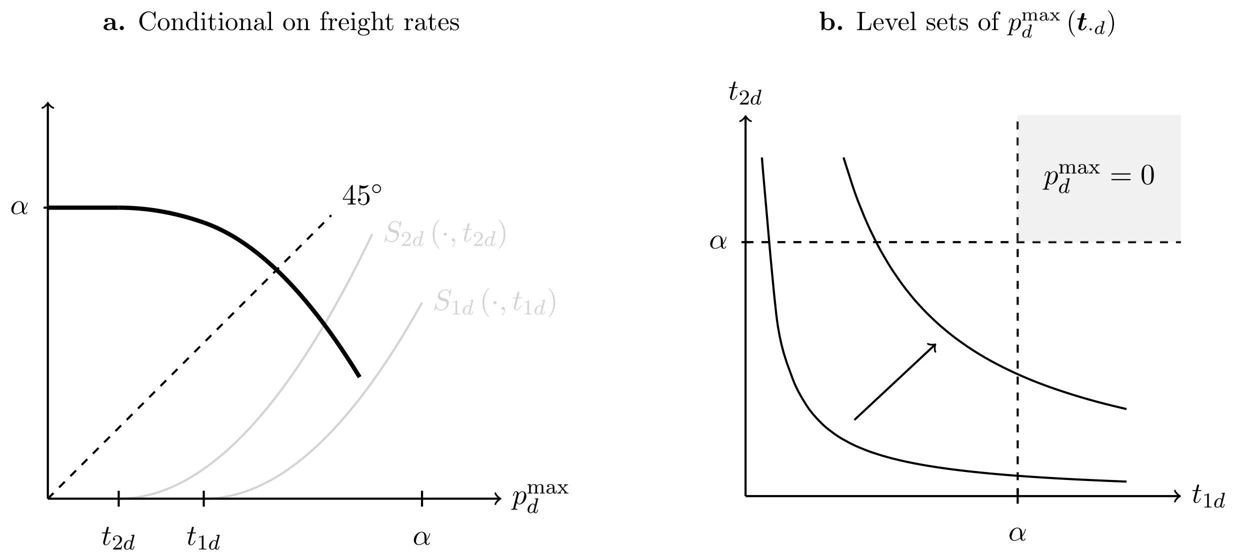
$$\begin{aligned} p^{\max}_{d} &= \alpha - \frac{\eta}{L_d} \sum_{o} S_{od}\left(p^{\max}_{d}, t_{od}\right) \\ &= \alpha - \frac{\eta}{2\gamma} \sum_{o} \int_{0}^{\hat{a}_{od}\left(p^{\max}_{d}, t_{od}\right)} \left[p^{\max}_{d} - C_{od}\left(a, t_{od}\right)\right] L_o \, \mathrm{d}G_o\left(a\right) \end{aligned} \tag{0.4}$$

where the second equality follows from substituting the profit-maximizing quantities (0.1).

[*] Main text

If $p_d^{\max} \leq t_{od}$, then even the most efficient o–based manufacturer prefers to stay out of the market, leading to zero sales from source o; $S_{od}(p_d^{\max}, t_{od}) = 0$. Note that the choke price is at least as great as the lowest freight rate. Otherwise, $p_d^{\max} \leq \min_o t_{od}$ implies that no manufacturers serve the market in question; the aggregate quantity across all origins, $\sum_o S_{od}(p_d^{\max}, t_{od})$, evaluates to zero. Therefore, *holding freight rates fixed,* an increase in the choke price weakly increases aggregate exports from any given source, so that $\sum_o S_{od}(p_d^{\max}, t_{od})$ is weakly increasing in $p_d^{\max}$. The RHS thus crosses the 45-degree line exactly once, which guarantees a unique solution, $p_d^{\max}(\boldsymbol{t}_{\cdot d})$, to (0.4).

Figure 1: Choke price, conditional on freight rates



Notes: **Panel a** Equilibrium choke price, given freight rates, as determined in (0.4). The light increasing lines plot $S_{od}(p_d^{\max}, t_{od})$ for $o = 1, 2$. Exports from o to d are zero if the relevant freight rate, t_{od}, exceeds the choke price. Otherwise, $S_{od}(\cdot, t_{od})$ is strictly increasing (recall that $p_d^{\max} \leq \alpha$). The dark downward sloping line displays $\alpha - (\eta/L_d)\sum_o S_{od}(p_d^{\max}, t_{od})$, the right-hand-side of (0.4). Its intersection with the 45-degree line gives the choke price conditional on freight rates, $p_d^{\max}(\boldsymbol{t}_{\cdot d})$. **Panel b** displays equilibrium choke price for any (t_{1d}, t_{2d}) pair. The choke price is effectively zero if $\min_o \{t_{od}\} > \alpha$.

Thus, the choke price is higher (and competition among manufacturers less fierce) when freight rates are high. When $p_d^{\max}(\boldsymbol{t}_{\cdot d})$ is differentiable,

$$\frac{\partial p_d^{\max}(\boldsymbol{t}_{\cdot d})}{\partial t_{od}} = \frac{\eta\, N_{od}(\boldsymbol{t}_{\cdot d})}{2\gamma + \sum_{o' \in \mathcal{O}} \eta\, N_{o'd}(\boldsymbol{t}_{\cdot d})} \in [0, 1), \tag{0.5}$$

where

$$N_{od}(\boldsymbol{t}_{\cdot d}) \equiv \widetilde{N}_o G_o(\hat{a}_{od}^*(\boldsymbol{t}_{\cdot d})) \tag{0.6}$$

is the mass of o–based exporters active in d. The choke price is more sensitive to freight rates in countries with a large mass of sellers in the destination in question. This, in turn, may be because there is a large number of (exogenous) potential entrants, or because the country is particularly good at producing the final good.

The insights from (0.5) allow us to complete the chain from freight rates to selection into exporting. Specifically, an increase in the freight rate $t_{o'd}$ affects the marginal o–based exporter if both o and o' export to d. If freight rates along (o, d) are high enough to thwart exports, then a marginal increase in $t_{o'd}$ does not affect entry. By the same token, if freight rates along (o', d) are so high as to choke off sales to the destination in question, then a further increase leaves the situation unchanged. I ignore these two cases by assuming that all countries export to d. Substituting (0.5) into (0.2), the extensive margin of exports in o depend on freight rates as

$$\frac{\partial \hat{a}^*_{od}\left(\boldsymbol{t}_{\cdot d}\right)}{\partial t_{o'd}} = \begin{cases} \dfrac{1}{w_o \tau_{od}} \dfrac{\partial p^{\max}_d \left(\boldsymbol{t}_{\cdot d}\right)}{\partial t_{o'd}} & \in \dfrac{1}{w_o \tau_{od}}[0, 1) \qquad \text{if } o' \neq o \\ \dfrac{1}{w_o \tau_{od}} \left(\dfrac{\partial p^{\max}_d \left(\boldsymbol{t}_{\cdot d}\right)}{\partial t_{od}} - 1 \right) & \in \dfrac{1}{w_o \tau_{od}}[-1, 0) \qquad \text{if } o' = o. \end{cases} \tag{0.7}$$

The mass of exporters from any given origin is therefore decreasing in the the own-freight rate, and increasing in cross-rates.

Effects of freight rate changes on demand for shipping

Equation (0.3) suggests that aggregate demand for shipping along route (o, d) depend on the the freight rates along routes destined for d, with cross-market effects mediated by the choke price, (0.5). I explore this relationship further by writing total shipping demand along route (o, d) as the product of the mass of o–based varieties sold in d and demand from the average surviving manufacturer,

$$S_{od}\left(\boldsymbol{t}_{\cdot d}\right) = N_{od}\left(\boldsymbol{t}_{\cdot d}\right) \times \overline{s}^*_{od}\left(\boldsymbol{t}_{\cdot d}\right).$$

Following Head and Mayer (2014), I decompose changes in log demand due to changes in freight rate $t_{o'd}$ into the extensive and intensive-and-compositional margins,

$$\frac{\partial \ln S_{od}\left(\boldsymbol{t}_{\cdot d}\right)}{\partial t_{o'd}} = \underbrace{\frac{\partial \ln N_{od}\left(\boldsymbol{t}_{\cdot d}\right)}{\partial t_{o'd}}}_{\text{extensive}} + \underbrace{\frac{\partial \ln \overline{s}^*_{od}\left(\boldsymbol{t}_{\cdot d}\right)}{\partial t_{o'd}}}_{\text{intensive + compositional}}. \tag{0.8}$$

The linearity of shipping demand in a implies that average shipping demand coincides with demand from the average manufacturer,

$$\mathbb{E}\left[s^*_{od}\left(a | \boldsymbol{t}_{\cdot d}\right) \middle| a \leq \hat{a}^*_{od}\left(\boldsymbol{t}_{\cdot d}\right) \right] = \overline{s}^*_{od}\left(\boldsymbol{t}_{\cdot d}\right),$$

so that the second term in (0.8) simplifies to

$$\frac{\partial \ln \overline{s}^*_{od}(\boldsymbol{t}_{\cdot d})}{\partial t_{o'd}} = \underbrace{t_{o'd}\frac{\mathbb{E}\left[\partial s^*_{od}(a|\,\boldsymbol{t}_{\cdot d})/\partial t_{o'd}|\,a\leq \hat{a}^*_{od}(\boldsymbol{t}_{\cdot d})\right]}{\overline{s}^*_{od}(\boldsymbol{t}_{\cdot d})}}_{\text{intensive}} + \underbrace{t_{o'd}\frac{\partial \hat{a}^*_{od}(\boldsymbol{t}_{\cdot d})}{\partial t_{o'd}}\frac{g_o(\hat{a}^*_{od}(\boldsymbol{t}_{\cdot d}))}{G_o(\hat{a}^*_{od}(\boldsymbol{t}_{\cdot d}))}\left(\frac{\hat{s}^*_{od}(\boldsymbol{t}_{\cdot d})}{\overline{s}^*_{od}(\boldsymbol{t}_{\cdot d})}-1\right)}_{\text{compositional}}. \tag{0.9}$$

Maintaining the assumption that both o and o' export to d,

$$\frac{\partial S_{od}(\boldsymbol{t}_{\cdot d})}{\partial t_{o'd}} = \begin{cases} \dfrac{L_d}{2\gamma}\dfrac{\eta\, N_{o'd}(\boldsymbol{t}_{\cdot d})}{2\gamma+\sum_l \eta\, N_{ld}(\boldsymbol{t}_{\cdot d})}N_{od}(\boldsymbol{t}_{\cdot d}) & \text{if } o'\neq o \\ -\dfrac{L_d}{2\gamma}\dfrac{2\gamma+\sum_{i\neq o}\eta\, N_{id}(\boldsymbol{t}_{\cdot d})}{2\gamma+\sum_l \eta\, N_{ld}(\boldsymbol{t}_{\cdot d})}N_{od}(\boldsymbol{t}_{\cdot d}) & \text{if } o'=o, \end{cases} \tag{0.10}$$

where I substitute (0.5) and (0.7). Demand is downward sloping in the own freight rate, and increasing in cross-rates. Further,

$$\frac{\partial S_{od}(\boldsymbol{t}_{\cdot d})}{\partial t_{o'd}} = \frac{\partial S_{o'd}(\boldsymbol{t}_{\cdot d})}{\partial t_{od}},$$

mirroring the symmetry of *inverse* demand highlighted in the main text.

Invertibility of demand system

Cournot competition requires a well-defined inverse demand system. Since it is fruitless to attempt to invert the demand system for freight rates that map to zero demand, I restrict attention to

$$\begin{aligned} T^*_d &\equiv \{\boldsymbol{t}_{\cdot d}:\ S_{od}(\boldsymbol{t}_{\cdot d})>0 \text{ for all } o\in\mathcal{O}\} \\ &= \{\boldsymbol{t}_{\cdot d}:\ \hat{a}^*_{od}(\boldsymbol{t}_{\cdot d})>0 \text{ for all } o\in\mathcal{O}\}, \\ &= \{(t_{1d},\ldots,t_{Od}):\ t_{od}<p^{\max}_d(t_{1d},\ldots,t_{Od}) \text{ for all } o\in\mathcal{O}\}, \end{aligned}$$

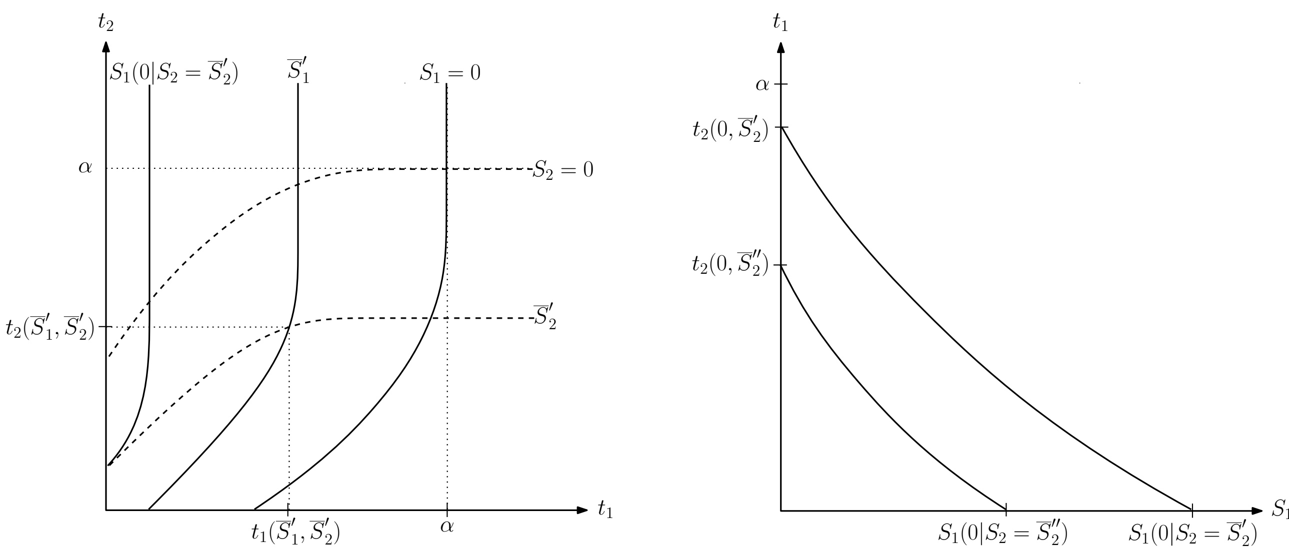
the set of freight rates consistent with strictly positive demand from all countries. To reach the third line, note that – holding freight rates in other countries fixed – demand along (o,d) is zero, $\hat{a}^*_{od}(\boldsymbol{t}_{\cdot d})=0$, if and only if t_{od} exceeds the equilibrium choke price $p^{\max}_d(\boldsymbol{t})$. In other words, demand is invertible only if freight rates are low enough manufacturers export from each country.

In the two–country case portrayed in Figure 2, T^*_d is the non-rectangular region enclosed by the $S_1=0$ and $S_2=0$ lines. This precludes applying the results in Cheng (1985) and Okuguchi (1987), which hold when T^*_d is rectangular, i.e., the Cartesian product of O intervals.

The following argument allows us to invert the demand system, at least in the two-country case. The inverse demands at the demand levels $(S_1,S_2)=\left(\overline{S}'_1,\overline{S}'_2\right)$ in Figure 2 is given by the

Figure 2: Demand and inverse demand for shipping

a. Levels sets of demand in 2 countries **b.** Inverse demand in country 1



Notes: **Panel a** displays level sets of $S_o(t_1, t_2)$, demands in countries 1 (solid) and 2 (dashed). Levels are $\overline{S}_o'' > \overline{S}_o' > 0$. The $S_o = 0$ line separates regions in (t_1, t_2) space with positive demand from country $o = 1, 2$ from those with zero demand. Demand for shipping from country o is weakly decreasing in the own rate t_o, and weakly increasing in the cross rate t_{-o}. **Panel b** displays inverse demand in country 1, conditional on country 2 demand.

intersection of the corresponding level sets, and are denoted by $t_o\left(\overline{S}_1', \overline{S}_2'\right)$. It is clear from the figure that this willingness-to-pay is weakly decreasing in either argument. Note that, as a special case (when $\overline{S}_o' = 0$), the intersection gives the choke price in country o. Finally, the greatest demand for shipping from, say, country 1, conditional on demand $\overline{S}_2'$ in country 2 is given by the level set through the intersection of the $\overline{S}_2'$-level set and the $t_1 = 0$ axis. Proceeding in this manner, we obtain the system of inverse demand functions $t_o(\boldsymbol{S}) \equiv t_o(S_1, S_2)$ for $o = 1, 2$.

For more than two countries, I appeal to Berry, Gandhi, and Haile (2013), who provide sufficient conditions for invertibility of the demand system $\{S_{od}(\boldsymbol{t}_{\cdot d})\}_o$. As luck would have it, their results apply even when T_d^* is non-rectangular. Berry et al. (2013) define an artificial country 0 with demand

$$S_{0d}(\boldsymbol{t}_{\cdot d}) \equiv 1 - \sum_{o \in \mathcal{O}} S_{od}(\boldsymbol{t}_{\cdot d}) = 1 - \frac{1}{\eta}\left(\alpha - p_d^{\max}(\boldsymbol{t}_{\cdot d})\right). \quad (0.11)$$

The demand system yields inverse demands, $\{t_{od}(\boldsymbol{S}_{\cdot d})\}_o$, if conditions (C1) and (C2) hold.

C1. Weak substitutability $S_{od}(\boldsymbol{t}_{\cdot d})$ is weakly increasing in $t_{o'd}$ for all $o \in \mathcal{O} \cup \{0\}$, and all $o' \in \mathcal{O} \setminus \{o\}$.

Proof. For all but the artificial country, we know from (0.3) that cross-market price effects are mediated by the choke price. The choke price operates along both the extensive margin (allowing more manufacturers to export when $p_d^{\max}$ is large), and along the intensive margin (allowing active manufacturers to charge to sell more). As a result, an increase in $p_d^{\max}$ unambiguously raises aggregate demand $S_{od}(\boldsymbol{t}_{\cdot d})$. From (0.5), the choke price $p_d^{\max}(\boldsymbol{t}_{\cdot d})$ is weakly increasing in each $t_{o'd}$, for $o' \in \mathcal{O}\setminus\{o\}$, so that $S_{od}(\boldsymbol{t}_{\cdot d})$ is weakly increasing in $t_{o'd}$ for all $o \in \mathcal{O}$, and all $o' \in \mathcal{O}\setminus\{o\}$.

As for the artificial country, we see from (0.11) that demand is weakly increasing in $t_{o'd}$, for $o' \in \mathcal{O}$, if and only if the equilibrium choke price, $p_d^{\max}(\boldsymbol{t}_{\cdot d})$, is increasing in $t_{o'd}$. □

C2. Connected substitution* The Jacobian matrix $\left[\partial S_{od}(\boldsymbol{t}_{\cdot d})/\partial t_{o'd}\right]_{o,o'}$, whose entries are given by (0.10), is invertible on T_d^*.

Proof. By assumption, all countries export to d on T_d^*. The determinant of the (symmetric) Jacobian matrix is

$$\det\left[\partial S_{od}(\boldsymbol{t}_{\cdot d})/\partial t_{o'd}\right] = \left(-\frac{L_d}{2\gamma}\right)^O \frac{2\gamma}{2\gamma + \sum_o \eta\, N_{od}} \prod_o N_{od},$$

which is nonzero under the maintained assumption that each country exports, $N_{od} > 0$ for all $o \in \mathcal{O}$. □

References

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